

came more than once. They really were very, very positive about it. The response has been fantastic.

JR: *What impact would you like the play to have, either on people in general or on the mathematical community?*

GC: It does seem to have an impact. It stirs people up, it gets them talking and thinking about issues about women and math and science. That's lovely, and I'm so glad that that's happened and that I can be of service in that way.

But that wasn't my intention exactly. I was acting purely as an artist. I had something to say and I wanted to say it in the most artistically satisfying way possible. I didn't really have an agenda or a moral or something I wanted to impart. I just knew I had to say what I had to say.

When I was initially working on this play, I immersed myself in a whole bunch of the most well-received, greatest, most celebrated solo shows that I could. The thing about solos is that when one person morphs into all these characters, if it's done well, the audience comes away with a sense of

our shared humanity, how we might not be all that different after all. There's something very deeply moving about that.

I hope that people come away from my show with that experience. I also hope that they laugh and have a good time, that they are moved and touched by it, and especially that they might think a little differently about what it's like to be a woman in math and science than when they sat down at the beginning of the show. Sometimes, you have to have a vicarious experience of someone in a certain situation before you really get it. You can talk about it in academic colloquia all day long, but sometimes it's that visceral thing you get from storytelling and theater art that kind of opens your eyes. I think maybe Larry Summers needs that.

Ms. De Cari is bringing *Truth Values* to universities and stages around the country. To discuss booking her show, contact booking@unexpectedtheatre.org.

Book Review

The Calculus of Friendship

Reviewed by Lawrence S. Braden

The Calculus of Friendship: What a Teacher and a Student Learned about Life While Corresponding about Math

Steven Strogatz

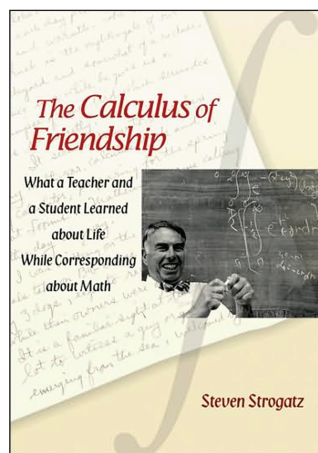
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Ostensibly, *The Calculus of Friendship* would at first appear to be a simple story of a high school student and his teacher, drawn together by their love of calculus, in which calculus was the bond that cemented their friendship over three decades. It is that, but so much more. Perhaps the book should be titled *A Calculus of Friendship*, using the tertiary definition of *calculus* as a recipe, or formula. But let Strogatz tell it in his own words: "Like calculus itself, this book is an exploration

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of change. It's about the transformation that takes place in a student's heart, as he and his teacher reverse roles, as they age, as they are buffeted by life itself. Through all these changes, they are bound together by a love of calculus. For them it is more than a science. It is a game they love playing together—

so often the basis of friendship between men—a constant while all around them is in flux" (from the Prologue, page xii).

Most readers would assume at first glance that the young math-geek Strogatz would be intellectually drawn to the subject by the older (by thirty years) Mr. Don Joffray, a physically imposing man, "a stronger version of Lee Marvin, whom I'd seen in lots of war movies." (Indeed, Joffray was at one

time the national whitewater kayaking champion). The truth is much more subtle and interesting. In fact Strogatz thought of Joffray as “soft”, not giving his students the red meat of mathematics. While the tenth grader Strogatz was learning the epsilon-delta definition of “continuity” in precalculus from his MIT-trained teacher Mr. Johnson, he found out that Joffray was telling his students that “a function is continuous if you can draw it without lifting your pencil from the paper.” (Lest we judge Mr. Joffray too harshly for this, none other than Creighton Buck once told me that in his freshman calculus classes at Wisconsin, he did not make much of an issue of epsilons and deltas either.) Don Joffray always wished that he knew more mathematics (who of us has not?). Thus Strogatz’s delicious morsels were a balm unto his soul and the beginning of a beautiful friendship.

I am exactly fifteen years older than Strogatz and fifteen years younger than Joffray and have taught calculus to bright high school students for thirty-five years in private schools in Hawaii and New England. Philosophically, I am more of a Mr. Johnson than a Mr. Joffray, perhaps to my students’ detriment, perhaps not. Who can say? But we all love calculus and all love sharing the wonderment of such a subject with others.

Joffray was probably learning as much about calculus from Strogatz and some of his other gifted students as they were learning from him. Joffray was the sort of teacher who would pose a problem to the class to which he did not know the answer. He took the greatest delight when one of his students would solve a problem such as this, especially when he was unable to solve it himself. So from the first the two of them were aware that the student was by far the more mathematically powerful of the pair and knew almost as much mathematics as the teacher.

But Joffray, a happy, enthusiastic man, challenged his students to a higher level than many more-traditional teachers. There was simply no telling what the “lesson of the day” would be. One day Joffray walked into the class and told of a goat he saw tethered to a tree that weekend. The goat, wishing to escape, went around and around the tree, keeping the rope taut. What was the equation of the goat’s path? Young Strogatz could learn all the calculus from the book himself. But it was questions such as these that hooked him. Indeed, he says on page 5, “With the passage of time I see now that I was like the goat tethered to the tree—and Mr. Joffray was the tree. I pulled taut on the rope and tried to get away from him, but only ended up wrapping myself closer and closer to him, all these years. How did that happen? It wasn’t because he taught me so much in the usual sense. No, his approach was so humble and unconventional, it confused me. It made me feel superior to him. I’m embarrassed to admit that, but it’s true.”

At the end of Strogatz’s junior year, his school (Loomis) held its annual awards ceremony. Strogatz won the Rensselaer Prize for the top junior in math and science. Joffray made a speech, likening his student to a mountain climber, ascending mathematical peaks and then returning with what he had seen. He made Strogatz feel generous, and even heroic.

The bonding started much more slowly than one might have thought. It was the student (perhaps once every year or so) writing to his old teacher and telling him about interesting problems he had come across in his college studies. One of the first concerned the irrationality of the square root of 2 and the proof that every precalculus high school student is taught—a very neat and clever proof, but one rooted in number theory. At Princeton Strogatz was taught a *geometrical* proof by his teacher, Benedict Gross. It was much more complicated than the standard one, but purely geometrical and very clever. After sharing it with Joffray, he challenged Joffray to use the method to prove that the sides of the Golden Rectangle are also incommensurable.

The chapters of *The Calculus of Friendship* are sewn together with an underpinning of advanced mathematics. As the friendship slowly(!) blossoms, the author uses specific mathematical problems they shared as chapter headings. (Appropriately, the chapter containing the problem about the square root of 2 is entitled “Irrationality”.) Some of the others, for a flavor of the book, are “Shifts”, “The Monk and the Mountain”, “Pursuit”, “Randomness”, “Infinity and Limits”, “Chaos” (of course!), and “Celebration”. Each of these is annotated, in chronological order, with the appropriate years when their relationship went through that phase. For instance, “The Monk and the Mountain” refers to the June 1961 column of *Scientific American* in which Martin Gardner posed the riddle of the monk who starts ascending a mountain at dawn, reaching the summit at dusk. He spends the night and, at dawn, starts down the mountain on the same spiraling path, reaching the bottom sometime before dusk. Every topologist knows that there is a point on the path where the monk must be at the same time each day. Perhaps this is the first “fixed point theorem” a student encounters, beautifully solved by imagining that there are *two* monks on the path the same day...one ascending and one descending. Surely they must meet. Strogatz likens this phase of their relationship (1989-1990) to the monk and the mountain. His own career as a teacher was about to take off at the Massachusetts Institute of Technology, and Joffray’s, at age sixty-one or so, reached its apex and was headed for decline. It was at that time that Strogatz and Loomis classmate Ed Rak, a mathematical genius, were invited to return to Loomis to give a speech at a faculty banquet honoring their old mentor.

Joffray had won the 1990 Swan Award for Teaching, given by a *consortium* of schools “to a teacher whose achievement represents the highest standards of the profession.” It was at this point that their correspondence really took off.

And thus it started, neither party imagining just where this correspondence would lead. Strogatz is not only a first-rate mathematician but a master teacher as well, as anyone who has seen him teach will testify. (I daresay that any person teaching high school mathematics should see his twenty-four lectures on chaos from The Teaching Company on DVD. They are masterpieces of clarity, and any inquisitive person, no matter what his or her background, would probably be enthralled with the subject.) Strogatz is the sort of person who not only enjoys learning his subject but enjoys sharing that learning with others, and who better than Mr. Joffray? The problems that Strogatz shared with Joffray, and his comments and solutions, are truly beautiful ones, most certainly more advanced than one usually is likely to run across in the Advanced Placement Calculus curriculum.

As an example, much emphasis is placed on *convergence* (absolute, conditional, or none) of infinite series in high school calculus. Few of these can actually be summed (ignoring specific ones obtained by “plugging in” to a Maclaurin series.) But what on earth is to be made of the infinite sum of $(\sin j)/j$, summing over the positive integers? Joffray asked Strogatz about this problem, which arose in his class. (Regrettably, the integral test, a standard tool, is invalid here.) What followed was a beautiful nine-page exposition into Fourier series, and the problem was solved. (Okay, if you are curious, it sums to $(\pi-1)/2$.)

Strogatz was so jazzed (his word) by his back-and-forth communications concerning this with his old teacher that he shared it with pal Rennie Mirollo while waiting for their food in a Chinese restaurant. Mirollo, with the help of the place mat, solved it using not Fourier series but Taylor series in the complex plane. A totally different and elegant solution! Joffray was ecstatic (there is no other word for it) upon receiving the second proof. This book is *chock full* of elegant problems, elegant solutions, and the give-and-take between two master teachers.

But something was missing from all of this. After years of corresponding with one another, Strogatz’s wife said “Well, after all this time, you two probably know quite a lot about each other.” Then it hit him...he did not really know his old teacher *at all*, not as a *person*. “You don’t!” she replied. “That is really such a *guy* thing!” Strogatz had, without ever realizing it, relegated Joffray to a category of “friendship” shared with his weekly basketball group—e.g., “how many children they had” was irrelevant. Strogatz came to realize, much to his shame, that it was he who had taken

refuge in the mathematics, shutting the door when Joffray talked about the loss of his twenty-seven-year-old son to cancer, or when Joffray mentioned Strogatz’s impending marriage. Life may be continuous, but it is certainly bound to contain non-differentiable points. On his side, Strogatz never mentioned that his marriage had failed or that his father had died. For a while Joffray’s letters lay unopened on Strogatz’s desk...he had stopped corresponding altogether. He at that stage of his life was emotionally exhausted. He didn’t even respond when Joffray wrote to say that he had suffered a stroke and was partially blind. In April 2004 Strogatz’s brother unexpectedly died. Joffray wrote to say that he had heard the news via the Loomis alumni magazine and sent condolences. It was this that that shook Strogatz out of his lethargy. He phoned his old mentor and said that he wanted to come up and, for the first time, *really* have a down-to-earth talk. With a tape recorder.

The rest is history. And a beautiful book, bound to become a classic in the mathematical literature, in this reviewer’s opinion. Like Hardy’s *A Mathematician’s Apology*, you do not have to know any mathematics whatsoever to read this book. It is a candid and all-too-human story told with brutal honesty...warts and all, sharing with the reader the elation and sincere regrets bound up in the relationship—but in the end, the victories, too. With some beautiful mathematics throughout!

Strogatz’s friend Alan Alda (yes, *that* Alan Alda) conducted a very perspicuous interview with the author. It may be accessed at <http://press.princeton.edu/video/strogatz/alda.html>.

But you would be cheating yourself if you only watched the interview and did not read the book.